Nonlinear Observer for P.E.M. Fuel Cells using a Takagi-Sugeno approach with unmeasurable premise variables*

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***Abstract*— In this paper, a nonlinear observer based on a Takagi Sugeno approach is developed and applied to a PEMFCS (Proton Exchange Membrane Fuel Cell Stack) in the case of unmeasurable premise variables. The FC model focuses on the auxiliary elements associated to the stack. The challenge is to construct a TS fuzzy observer such that it can asymptotically estimate the states of the considered nonlinear systems despite the unmeasurable premise variables. The results are validated through the use of the professional software AMESim using a co-simulation with Matlab-Simulink.**

I. INTRODUCTION

Fuel Cell Systems (FCS) usage has grown in the last decade, in-depth research being done in order to reduce the costs of the devices and the hydrogen fuel. From all the different types of FCS, the Proton Exchange Membrane Fuel Cell (PEMFC) system positioned itself as one of the most promising candidates to substitute traditional systems such as internal combustion engines. The PEMFC is an attractive candidate not only for transportation systems but also for other applications such as stationary systems and portable systems due to its high power density and low operating temperature compared to other types of fuel cells.

Fuel Cells represent interesting processes from a systemic point of view, presenting a high degree of nonlinearity, different scales in parameters and also variable dynamics (from mechanical/electrical/chemical equations). The need for diagnostics [2] and control [14] makes the construction of state observers a requirement. In this regard, the model based approach brings precision and a higher degree of generality but it comes at the price of difficulty in equations modeling and the risk of over complicating the system.

The design of state observers for nonlinear systems using Takagi-Sugeno (TS) models has been actively considered during the last decades ([4]-[9]). TS models are currently being used for a large class of physical and industrial processes. A TS model is a nonlinear model composed of multiple linear ones blended together with specialized nonlinear functions. Therefore the use of membership functions is employed to describe the fuzzy summation of all the linear systems by means of rules that give a precise representation of the nonlinear model in a compact state space. In addition, the structure of TS fuzzy models allows the use of well established tools from control system theory that have been developed over many years[3].

The Takagi-Sugeno fuzzy observer presents advantages over other types of observers because of its flexibility regarding nonlinear systems and its simplicity in construction and manipulation. The general form adopted by most of the authors extends the Luenberger observer and the unknown input observer to nonlinear systems.

Many works consider the case of TS models with measurable premise variables such as [10], [11]. This choice provides some simplicity, being an extension of classical algorithms developed for linear systems over the TS model. Unfortunately, the premise variables in practice are unmeasurable and the problem of estimation becomes more difficult to resolve. Different approaches have been adopted in order to solve the unmeasurable premise variable case[12], each trying to reduce as much as possible the restrictions that grow significantly with the system's complexity.

In this work, a physical model of a PEMFC is presented based on [1] and transformed into a TS fuzzy model. The systematic methodology for model construction is performed with the aid of a sector nonlinearity approach [8]. In order to estimate the states, a Takagi-Sugeno observer proposed in [5] is used. The challenge is to construct an observer such that it can asymptotically estimate the states of the considered nonlinear systems. The measured premise variable case, applied to PEMFCS, has been studied in a previous work [13]. Few other works have applied to a Fuel Cell System [15].

The state estimations method based on nonlinear observer is evaluated in simulation on a PEMFC running on AMESim platform in co-simulation with Mathworks' Matlab.

The contribution of this paper is:

- The validation of the nonlinear physical Fuel Cell model and the Takagi Sugeno Model using AMESim Software.

- Obtaining a Takagi Sugeno Observer with unmeasurable premise variables for it.

This paper is organized as follows. In Section II, the PEM Fuel cell with auxiliary elements is described, the nonlinear model is explained followed by a model validation by AMESim software. In Section III the model is transformed into a TS model and the nonlinear TS observer is presented. Sections IV deals with the state estimations where the gains of the observer are computed by making use of Linear Matrix Inequalities. Conclusions and future work are finally presented in the last section.

II. Fuel Cell Modeling

A. State Space modeling Modeling

The current work is focused on a Polymer Electrolyte Membrane (Proton Exchange Membrane) Fuel Cell Stack that uses a Nafion 117 membrane, including auxiliary components (Figure. 1). The model presents some simplifications by taking into consideration only pure Oxygen as input and also an ideal humidifying and cooling unit. Also spatial variations are ignored as well as temperature variations. The pressure difference between Anode and Cathode is maintained minimal by means of a regulator.

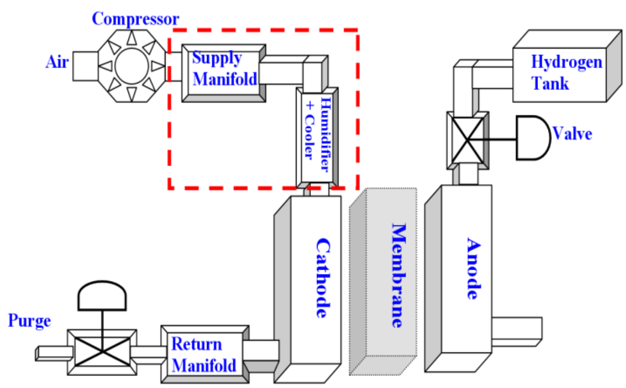

Figure 1. PEM Fuel Cell system with auxiliary elements attached

The system equations used are based on the work presented in [1].

Mass conservation law is applied for the case of ideal gases.

$$\begin{cases} \dfrac{dp_{sm}}{dt} = \dfrac{R_{O_2}.T_{st}}{V_{sm}} \cdot (W_{cp} - W_{sm,out}) \\ \dfrac{dp_{rm}}{dt} = \dfrac{R_a T_{rm}}{V_{rm}} \cdot \left(W_{ca,out} - W_{rm,out}\right) \\ \dfrac{dp_{O_2,ca}}{dt} = \dfrac{R_{O_2} \cdot T_{st}}{V_{ca}} \cdot \left(W_{O_2,ca,in} - W_{O_2,ca,out} - W_{O_2,reacted}\right) \\ \dfrac{dp_{v,ca}}{dt} = \dfrac{R_v \cdot T_{st}}{V_{ca}} \left(-W_{v,ca,out} + W_{v,ca,gen}\right) \\ \dfrac{dp_{H_2,an}}{dt} = \dfrac{R_{H_2} \cdot T_{st}}{V_{an}} \left(W_{H_2,an,in} - W_{H_2,an,out} - W_{H_2,reacted}\right) \end{cases} \tag{1}$$

The compressor's mass flow is considered as input to the system, existing the possibility to add a regulator so that it follows a certain flow.

In the cathode we have mass that accumulates, depending on the quantity of oxygen that enters, exits and reacts with the hydrogen ions. Also vapor is generated inside the cathode, some of it exiting towards the return manifold, while another part remaining inside the cathode adding to the general pressure. We have worked in the hypothesis that there is no humidification neither of the oxygen nor of the hydrogen.

Concerning the mass flow through valves, we have considered a simple linear model for the ones that would have small pressure differences; whereas a chocked or unchocked regime equation was required for the purge valves of the cathode and anode because of the big pressure difference created with the atmospheric pressure. Also we only considered the chocked equations, which indeed is not a restrictive assumption considering the fact that the return manifold pressures overpass 2 bar.

The electrical current refers to the demanded current by the consumer system attached to the fuel cell.

In the end, we reach the following dynamical equations:

$$\begin{cases} \dfrac{dp_v}{dt} = \dfrac{R_v \cdot T_{st}}{V_{ca}} \cdot \left[\left(k_{ca}\left(p_{ca,O_2} + p_v - p_{rm}\right) \cdot \left(-1 + \dfrac{M_{O_2} p_{ca,O_2}}{M_{O_2} p_{ca,O_2} + M_v p_v} \right) \right) + \left(\dfrac{n \cdot M_v}{2 \cdot F} \right) \cdot I_{st} \right] \\ \dfrac{dp_{ca,O_2}}{dt} = \dfrac{R_{O_2} \cdot T_{st}}{V_{ca}} \cdot \left[\begin{array}{l} k_{sm} \cdot \left(p_{sm} - p_{ca,O_2} - p_v\right) - k_{ca} \cdot \left(p_{ca,O_2} - \left(p_{rm} - p_v\right)\right) - \\ - \left(\dfrac{n \cdot M_{O_2}}{4 \cdot F} \right) \cdot I_{st} \end{array} \right] \\ \dfrac{dp_{rm}}{dt} = \dfrac{R_a \cdot T_{st}}{V_{rm}} \cdot \left[\begin{array}{l} k_{ca} \cdot \left(p_{ca,O_2} + p_v - p_{rm}\right) - \\ - \dfrac{A_{T,rm} \cdot C_{d,rm} \cdot p_{rm}}{\sqrt{\bar{R} \cdot T_{atm}}} \cdot \sqrt{\gamma} \cdot \left(\dfrac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \end{array} \right] \\ \dfrac{dp_{sm}}{dt} = \dfrac{R_a \cdot T_{st}}{V_{rm}} \cdot \left[W_{cp} - k_{sm} \cdot \left(p_{sm} - p_{ca,O_2} - p_v\right) \right] \\ \dfrac{dp_{an,H_2}}{dt} = \dfrac{R_{H2} T_{st}}{V_{an}} \cdot \left[\begin{array}{l} K \cdot K_1\left(p_{sm} - p_{an,H_2}\right) - \\ - A_{an} \cdot p_{an,H_2} \cdot \dfrac{C_{d,an} \cdot \sqrt{\gamma} \cdot \left(\dfrac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\sqrt{\bar{R} \cdot T_{atm}}} - M_{H_2} \cdot \dfrac{n \cdot I_{st}}{2 \cdot F} \end{array} \right] \end{cases} \tag{2}$$

B. Model validation using professional simulation software

The constructed model has been verified on a simulation platform, specifically *LMS AMESIM*. An equivalent model has been developed (Figure 2) and co-simulated with our state space model implemented in *Simulink*.

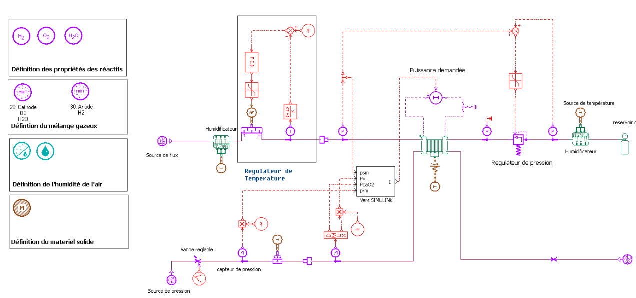

Figure 2. The equivalent AMESIM model of our system

The co-simulation capability refers to the interoperability between a Simulink and an AMESim simulation. The two software applications can exchange data during runtime giving AMESim the possibility to act as a real process interfaced to a monitoring or control system.

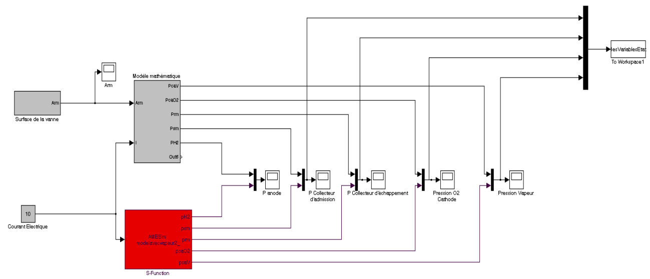

Figure 3. The schema reflecting the interconnections for the co-simulation

We can see some of the state values in the following graphics. The results obtained represent a comparison between the AMESim and Simulink values.

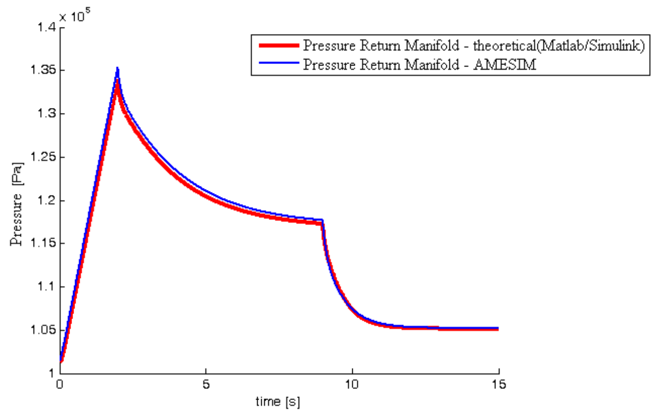

Figure 4. Return Manifold Pressure evolution in paralel AMESIM/Simulink

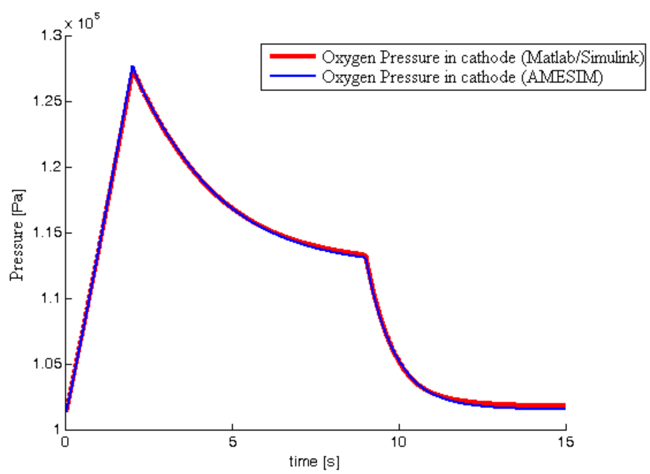

Figure 5. The Oxygen Pressure inside the cathode volume, simulation AMESIM/Simulink

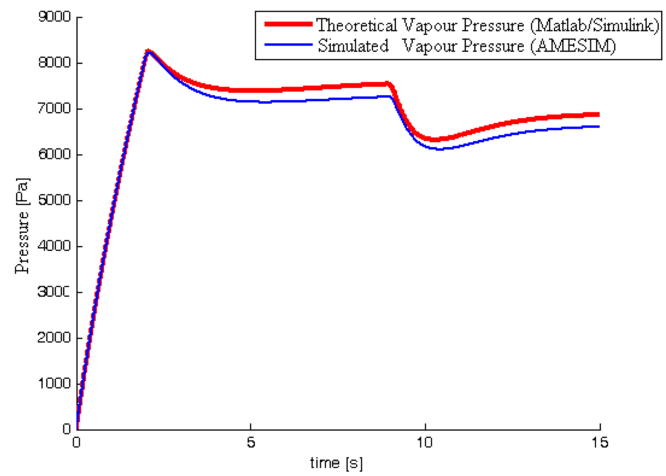

Figure 6. The vapour Pressure present inside the cathode volume

The tests were made for a predefined set of inputs. The powerful influence in 3 stages of the opening of the Return Manifold valve can be easily seen on all the states. The electrical current increases the generation of vapor. Here we can see a small difference caused by the equations utilized but it's negligible.

The Hydrogen and the Supply Manifold pressures weren't presented in order to limit the number of pages.

C. The Takagi-Sugeno Model

The Takagi-Sugeno model is in fact a multi-model, fuzzy logic system, that permits to write the general nonlinear state space system as a sum of linear systems normalized by an associated membership function(generally triangular membership functions are used). As such we obtain a new system:

$$\begin{cases} \dot{x} = \sum_{i=1}^{8} \left[w_i(x) \cdot \left(A_i \cdot x + B_i \cdot u \right) \right] \\ y = C \cdot x \end{cases} \quad (3)$$

Where $w_i(x)$ is a Membership function that returns the degree of validity of the ith linear model. It varies in the interval (0,1) in accordance to the nonlinear premise variables(nonlinear terms existent in our model). The matrices A_i, B_i form each of the linear system. One important fact is that the number of linear elements is equal to $2^{\text{number of premise variables}}$=8 for our case.

In order to develop the TS system, we first need to rewrite the initial state space model so that we separate the nonlinear terms.

$$\begin{cases} \dot{x} = A_x(x) \cdot x + B_x(x) \cdot u \\ y = C \cdot x \end{cases} \quad (4)$$

We separate the nonlinear terms into the 2 matrices

$A_x(x)$ *and* $B_x(x)$. For our system we have chosen 5 states(Vapor Pressure; Oxygen Pressure in Cathode; Return manifold Pressure; Supply Manifold Pressure; Hydrogen

Pressure in the Anode), with two of them, the Return Manifold pressure and the Supply Manifold Pressure, exiting directly as outputs.

$$x = \begin{pmatrix} p_v \\ p_{ca,O_2} \\ p_{rm} \\ p_{sm} \\ p_{an,H_2} \end{pmatrix} \quad u = \begin{pmatrix} I_{st} \\ A_{T,rm} \\ W_{in} \end{pmatrix} \quad y = \begin{pmatrix} p_{rm} \\ p_{sm} \end{pmatrix} \tag{5}$$

The way we choose the nonlinear terms is important, the objective being to keep their number to a minimum while maintaining the observability of each new linear system that will be constructed. Regarding the observability property, the condition that each linear system is observable is a requirement, but the observability is dependent on the minimum and maximum values chosen and also on the selection of the premise variables. So if one linear system is not observable this does not mean the whole nonlinear system is not observable. We just have to change the minimum and maximums or to select the premise variables differently (increasing or decreasing their number). Taking this into account and also the fact that, if one or more nonlinear terms participate in a linear operation it will not affect the way we choose the premise variable, we obtain:

$$B_x(x) = \begin{pmatrix} cst1 * \left[\frac{n \cdot M_v}{2 \cdot F}\right] & 0 & 0 \\ cst2 * \left[-\frac{n \cdot M_{O_2}}{4 \cdot F}\right] & 0 & 0 \\ 0 & -\frac{R_a \cdot T_{st}}{V_{rm}} \cdot \frac{C_{d,rm} \cdot \sqrt{\gamma} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\sqrt{R \cdot T_{atm}}} \cdot (p_{rm}) & 0 \\ 0 & 0 & \frac{R_a \cdot T_{st}}{V_{rm}} \\ -\frac{R_{H2} T_{st}}{V_{an}} M_{H_2} \cdot \frac{n}{2 \cdot F} & 0 & 0 \end{pmatrix} \tag{6}$$

$$A_x(x) = \begin{pmatrix} -k_{ca} \cdot cst1 & cst1 \cdot \left[-k_{ca} + k_{ca} \cdot Z_1(p_v, p_{ca,O_2}) + k_{ca} \cdot Z_2(p_v, p_{ca,O_2})\right] & cst1 \cdot \left[k_{ca} - k_{ca} \cdot Z_1(p_v, p_{ca,O_2})\right] & 0 & 0 \\ cst2 \cdot [-k_{sm} - k_{ca}] & cst2 \cdot [-k_{sm} - k_{ca}] & cst2 \cdot k_{ca} & 0 & 0 \\ cst3 \cdot k_{ca} & cst3 \cdot k_{ca} & -cst3 \cdot k_{ca} & 0 & 0 \\ cst3 \cdot k_{sm} & cst3 \cdot k_{sm} & 0 & -cst3 \cdot k_{sm} & 0 \\ 0 & 0 & 0 & cst5 & -cst6 - cst5 \end{pmatrix}$$

In conclusion we have found three premise variables:

$$\begin{cases} Z_1(p_v, p_{ca,O_2}) = \dfrac{M_{O_2} \cdot p_{ca,O_2}}{M_{O_2} \cdot p_{ca,O_2} + M_v \cdot p_v} \\ Z_2(p_v, p_{ca,O_2}) = \dfrac{M_{O_2} \cdot p_v}{M_{O_2} \cdot p_{ca,O_2} + M_v \cdot p_v} \\ Z_3(p_{rm}) = p_{rm} \end{cases} \tag{7}$$

In (7) we note the fact that a division by 0 is an impossible case, the Pressures being positive and greater than 0.

In order to construct the model there are different methods, but the most used is the Sector Nonlinearity approach. This method is based on finding local maximum and minimum values for the premise variables. In order to find these intervals, reference [6] presents a graphical method. This method is based on drawing a multidimensional graph of each premise variable in dependence with its forming states. Its only limit is the fact that it's restricted to premise variables dependant on maximum 3 states. Also the states are considered decoupled so the obtained graph needs to be segmented in sectors in accordance with the real states behavior tested experimentally.

III. TAKAGI-SUGENO OBSERVER

A. Observer design

Having the Takagi Sugeno model, we can now develop the observer.

For this, based on the theory presented in [9], we modify the TS system as follows:

$$\begin{cases} \dot{x} = \sum_{i=1}^{8} \left[\mathrm{w}_i(x) \cdot \left(A_0 \cdot x + \overline{A}_i \cdot x + B_i \cdot u \right) \right] \\ y = C \cdot x \end{cases} \tag{8}$$

$$\text{Where: } \begin{cases} A_i = \overline{A}_i + A_0 \\ A_0 = \dfrac{1}{8} \sum_{i=1}^{8} A_i \end{cases} \tag{9}$$

Thus we will choose the model of the observer as:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{8} \left[\mathrm{w}_i(\hat{x}) \cdot \left(A_0 \cdot \hat{x} + \overline{A}_i \cdot \hat{x} + B_i \cdot u + L_i \cdot (y - \hat{y}) \right) \right] \\ \hat{y} = C \cdot \hat{x} \end{cases} \tag{10}$$

The general procedure used for computing a TS system observer is to impose stability to the error dynamics of the estimation.

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} \tag{11}$$

By means of Lyapunov asymptotic stability theory we can obtain a set of Linear Matrix Inequalities (LMI) in order to calculate the gains of the observer matrix:

For i = 1,…,5

$$\begin{cases} A_0^T \cdot P + P \cdot A_0 - C^T \cdot K_i^T - K_i \cdot C < Q \\ \begin{bmatrix} \left[Q + \lambda_1 \cdot M_i^2 \cdot I_4 + \lambda_2 \cdot N_i^2 \cdot I_4 \right] & P \cdot \overline{A}_i & P \cdot B_i \\ \overline{A}_i \cdot P & -\lambda_1 \cdot M_i^2 \cdot I_4 & 0 \\ B_i^T \cdot P & 0 & -\lambda_2 \cdot N_i^2 \cdot I_3 \end{bmatrix} < 0 \\ Q < 0, symmetric \\ P > 0, symmetric \\ \lambda_1 > 0, scalar \\ \lambda_2 > 0, scalar \end{cases} \tag{12}$$

The observer gains are calculated afterwards as:

$$L_i = P^{-1} \cdot K_i \tag{13}$$

We need to add the required inequalities:

$$\begin{cases} \left| \mathrm{w}_i(x) \cdot \mathrm{x} - \mathrm{w}_i(\hat{x}) \cdot \hat{\mathrm{x}} \right| \le N_i \cdot |x - \hat{x}| \\ \left| \left(\mathrm{w}_i(x) - \mathrm{w}_i(\hat{x}) \right) \right| \le M_i \cdot |x - \hat{x}| \\ |u| \le \beta (\text{bounded input}) \end{cases} \tag{14}$$

In order to calculate N_i and M_i, as showed in [12], a Taylor expansion at order zero with an integral remainder term of f(x) around $\hat{x}$ gives us:

$$\begin{cases} |\mathrm{f}(x)\text{-}\mathrm{f}(\hat{x})| \le J \cdot |x-\hat{x}| \\ J = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \\ a_{ij} = \max_{t\in[x_j,\hat{x}_j]} \left| \dfrac{\partial f_i}{\partial x_j} \right| \end{cases} \tag{15}$$

So for calculating N_i, we just consider the above function as being multiplied by x respectively $\hat{x}$, before doing the partial derivatives when calculating the Jacobian matrix.

To calculate effectively the local Lipchitz constants needed, after finding M_i and N_i as matrices, a simple singular value decomposition would generated the required value, although the LMIs may work with M_i, N_i as constants or matrices.

Demonstration:

In order to build a demonstration for the LMIs previously mentioned we start just from replacing (8) and (10) in (11)=>

$$\dot{\tilde{x}} = \sum_{i=1}^{5} \left[\left(\overline{A}_i \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) + B_i \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u + (A_0 - L_i\cdot C)\cdot \tilde{x} \right) \right] \tag{16}$$

The theorem says that by choosing a Lyapunov candidate function (in this case, a basic quadratic one): $V = \tilde{x}^T \cdot P \cdot \tilde{x} > 0$, if we find a matrix $P = P^T > 0$ so that we will have $\dot{V} = \dot{\tilde{x}}^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot \dot{\tilde{x}} < 0$ then the estimation error is asymptotically stable.

Replacing (16) in the time derivative of the candidate function we obtain:

$$\left\{ \sum_{i=1}^{5} \begin{bmatrix} \left(\overline{A}_i \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) + B_i \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u \right) + \\ + (A_0 - L_i \cdot C)\cdot \tilde{x} \end{bmatrix} \right\}^T \cdot P \cdot \tilde{x} + \tag{17}$$
$$+ \tilde{x}^T \cdot P \cdot \left\{ \sum_{i=1}^{5} \begin{bmatrix} \left(\overline{A}_i \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) + B_i \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u \right) + \\ + (A_0 - L_i \cdot C)\cdot \tilde{x} \end{bmatrix} \right\} < 0$$

We can rearrange so that:

$$\sum_{i=1}^{5} \begin{bmatrix} (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}})^T \cdot \overline{A}_i^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot \overline{A}_i \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) + \\ + u^T \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))^T \cdot B_i^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot B_i \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u \end{bmatrix} + \tag{18}$$
$$+ \sum_{i=1}^{5} \left[\tilde{x}^T \cdot (A_0 - L_i \cdot C)^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot (A_0 - L_i \cdot C)\cdot \tilde{x} \right] < 0$$

As in the previously mentioned work [9] we can use the following Lemma:

For any X, Y matrices, and a positive scalar λ, we will always have:

$$X^T \cdot Y + Y^T \cdot X \le \lambda \cdot X^T \cdot X + \lambda^{-1} \cdot Y^T \cdot Y \tag{19}$$

By applying (19) to the terms inside the first sum in (18) we will arrive at the following system of inequalities:

For i = 1,…,5

$$\begin{cases} (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}})^T \cdot \overline{A}_i^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot \overline{A}_i \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) \le \\ \lambda_1 \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}})^T \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) + \lambda_1^{-1} \cdot \tilde{x}^T \cdot P \cdot \overline{A}_i \cdot \overline{A}_i^T \cdot P \cdot \tilde{x} \\ u^T \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))^T \cdot B_i^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot B_i \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u \le \\ \lambda_2 \cdot u^T \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))^T \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u + \lambda_2^{-1} \cdot \tilde{x}^T \cdot P \cdot B_i \cdot B_i^T \cdot P \cdot \tilde{x} \end{cases} \tag{20}$$

And by using (14) we get:

$$\begin{cases} (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}})^T \cdot \overline{A}_i^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot \overline{A}_i \cdot (\mathrm{w}_i(x)\cdot \mathrm{x}\text{-}\mathrm{w}_i(\hat{x})\cdot \hat{\mathrm{x}}) \le \\ \lambda_1 \cdot N_i^T \cdot N_i + \lambda_1^{-1} \cdot \tilde{x}^T \cdot P \cdot \overline{A}_i \cdot \overline{A}_i^T \cdot P \cdot \tilde{x} \\ u^T \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))^T \cdot B_i^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot B_i \cdot (\mathrm{w}_i(x)\text{-}\mathrm{w}_i(\hat{x}))\cdot u \le \\ \lambda_2 \cdot M_i^T \cdot M_i \cdot \beta^2 + \lambda_2^{-1} \cdot \tilde{x}^T \cdot P \cdot B_i \cdot B_i^T \cdot P \cdot \tilde{x} \end{cases} \tag{21}$$

So from (21) and if we consider each iteration in the sum in (14), we could write each as an inequality:

$$\begin{cases} \lambda_1 \cdot N_i^T \cdot N_i + \lambda_1^{-1} \cdot \tilde{x}^T \cdot P \cdot \overline{A}_i \cdot \overline{A}_i^T \cdot P \cdot \tilde{x} + \\ + \lambda_2 \cdot M_i^T \cdot M_i \cdot \beta^2 + \lambda_2^{-1} \cdot \tilde{x}^T \cdot P \cdot B_i \cdot B_i^T \cdot P \cdot \tilde{x} + \\ + \tilde{x}^T \cdot (A_0 - L_i \cdot C)^T \cdot P \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot (A_0 - L_i \cdot C)\cdot \tilde{x} < 0 \end{cases} \tag{22}$$

At this point we have obtained 8 Matrix inequalities in our case. Because we still cannot reach an LMI format, being unable to eliminate directly the errors and the inverses of the λ scalars, we modify (22). So by separating the last two terms in the inequality and by writing the Schur form for the other terms we obtain (12). Therefore, currently we have obtained conditions for asymptotic stability, but we have no conditions for imposing time response performances. For this we can see in [12] that we can try to force a faster response time by setting restrictions over the eigenvalues of the *(A_0 - L_iC)*. Conditions manifested through the constants $v_{\max}$, $v_{\min}$. So (12) becomes:

$$\begin{cases} \left(A_0^T \cdot P + P \cdot A_0 - C^T \cdot K_i^T - K_i \cdot C + 2 \cdot v_{\max} \cdot P \right) < Q \\ -\left(A_0^T \cdot P + P \cdot A_0 - C^T \cdot K_i^T - K_i \cdot C + 2 \cdot v_{\min} \cdot P \right) < Q \\ \begin{bmatrix} \left[Q + \lambda_1 \cdot M_i^2 \cdot I_4 + \lambda_2 \cdot N_i^2 \cdot I_4 \right] & P \cdot \overline{A}_i & P \cdot B_i \\ \overline{A}_i \cdot P & -\lambda_1 \cdot M_i^2 \cdot I_4 & 0 \\ B_i^T \cdot P & 0 & -\lambda_2 \cdot N_i^2 \cdot I_3 \end{bmatrix} < 0 \end{cases} \tag{23}$$

One observation we have to add is that if the system is very restrictive then we can separate the matrix LMI into 2 inequalities, one for terms containing M and A terms, while the other N and B terms. Also we replace Q with some matrices Q1 respectively Q2 for each of the new ones.

B. Observer results

By implementing the presented techniques, we obtained the following results.

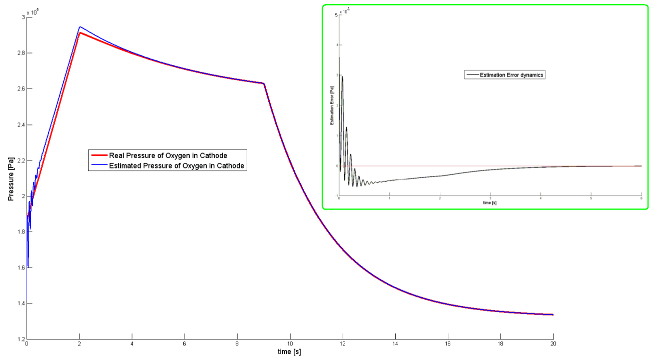

Figure 7. The evolution of the real and estimated values of the Oxygen Pressure in the cathode

In Figure 7 we have tested the performance of the Observer on the Oxygen Pressure inside the cathode with a different initial state than the system. We can see that the estimation error reaches stability with no static error. The stabilization requires a certain time because of the high sloped evolution.

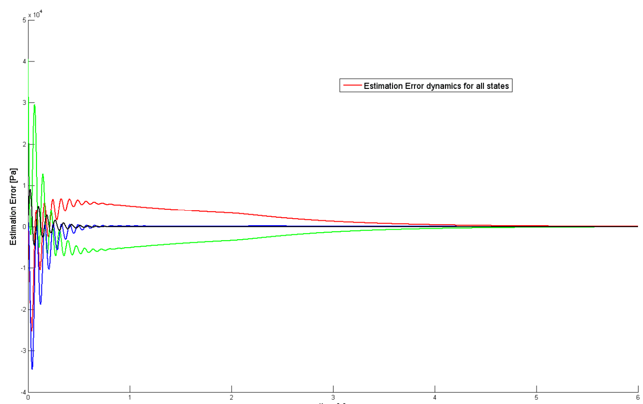

Figure 8. The evolution of the estimation error for all the states

Again in figure 8 we can see the powerful oscillatory behavior in the beginning of the simulation. This is caused by the existence of non minimum phase zeros in (A_0 - L_iC) that forces an undershoot and some consequent oscillations. Also regarding the time required for stabilization, this is due to the eigenvalues of the (A_0 - L_iC). As the values of $v_{\min}$ are chosen too close to -1 then the LMI doesn't find any solution. This could be caused by the restrictions generated in the construction of the LMIs or/and the minimum and maximum values chosen for the Jacobian.

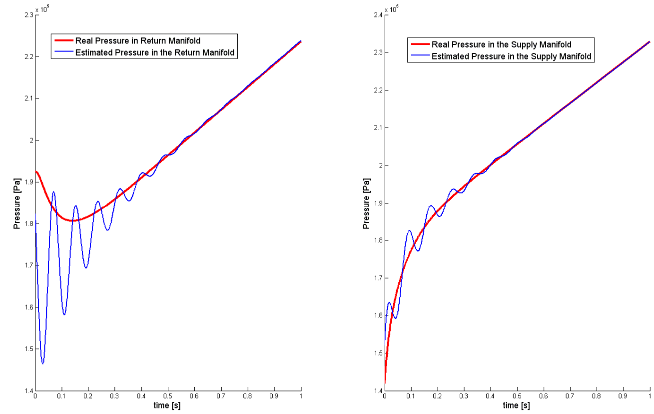

Figure 9. Stabilization of the estimation error, for Return Manifold/Supply Manifold Pressures

IV. Conclusion

The Takagi-Sugeno method is beginning to be more and more used both for control and diagnosis, and it is necessary for the approach to be applied in applications with an even greater complexity in order to find and improve the problems that TS observers may have; such drawbacks like: the exponential evolution of the complexity that comes with the increase of the number of premise variables; the sensibility towards large Lipchitz constants; difficulty in reducing the restrictions generated by the way the inequalities are established when the LMIs are formed. Also like most Lipchitz based methods, the associated constant may bring large estimation errors.

To conclude, the presented method is proven effective for a quite complex system, bringing into perspective the possibility to apply the method to an even more complex FC model, eventually a model used for diagnostic or control.

Appendix

		Variable	Description
η_{cp}	Compressor efficiency	*Ra*	Gas constant (J/mol.K)
$C_{D,rm}$	Discharge coefficient nozzle		
p_{sm}	Pressure supply manifold (Pa)	*R*	Gas constant (J/kg.K)
p_{rm}	Pressure return manifold (Pa)	T_{atm}	Air temperature (K)
w_{cp}	Compressor speed (rad/s)	T_{rm}	Temperature return manifold(K)
I_{st}	Current in the stack (A)	$T_{cp,out}$	Ai outputted temperature (K)
T_{st}	Temperature in the stack (K)	P_{atm}	Air pressure (Pa)
γ	specific heat capacity of gas		

		Variable	Description
V_{ca}	Cathode volume (m^3)	C_p	Specific heat capacity of air (J/kg.K)
V_{an}	Anode volume (m^3)	F	Faraday number
V_{rm}	Return manifold volume (m^3)	M_{02}	Molar masse of Oxygen (kg/mol)
V_{sm}	Supply manifold volume (m^3)	M_{H2}	Molar masse of Hydrogen(kg/mol)
W_{cp}	Compressor mass flow (kg/s)	$k_{sm,out}$	Supply manifold outlet flow constant (kg/s.Pa)
η_{cp}	Compressor efficiency	$k_{rm,out}$	Return manifold outlet flow constant (kg/s.Pa)
τ_{cm}, τ_{cp}	Torques of motor and compressor (N.m)	$A_{T,rm}$	Return manifold nozzle area (m^2)

		Variable	Description
R_v	Vapor gas constant	R_{H_2}	Hydrogen gas constant
R_{O_2}	Oxygen gas constant	$C_{d,an}$	Hydrogen purge nozzle discharge coefficient
M_v	Molar mass of vapor	$W_{sm,out}$	Mass flow exiting the supply manifold[kg/s]
k_{term}	Constants representing the linearization coefficient describing a valve[kg/(s*Pa)]	$W_{ca,out}$	Mass flow exiting the cathode[kg/s]
$W_{rm,out}$	Mass flow exiting the return manifold[kg/s]	$W_{O_2,ca,in}$	Mass flow of oxygen entering the cathode[kg/s]
$W_{H_2,reacted}$	Mass flow of oxygen reacted inside the anode[kg/s]	$\bar{R}$	Universal gas constant
$W_{O_2,reacted}$	Mass flow of oxygen in the cathode that reacts with the electrons and ions to form vapor	$W_{v,ca,gen}$	Mass flow of generated water inside the cathode

$$cst1 = \frac{R_v \cdot T_{st}}{V_{ca}};\ cst2 = \frac{R_{O_2} \cdot T_{st}}{V_{ca}};\ cst3 = \frac{R_a \cdot T_{st}}{V_{rm}};\ cst4 = \frac{M_v}{M_{O_2}};$$

$$cst5 = K \cdot K_1 \cdot \frac{R_{H_2} \cdot T_{st}}{V_{an}};\ cst6 = A_{an} \cdot \frac{C_{d,an} \cdot \sqrt{\gamma} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2\cdot(\gamma-1)}}}{\sqrt{\bar{R} \cdot T_{atm}}}$$

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